Motivated Prospects of Upward Mobility*

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Abstract

The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect upward mobility and fear that high redistribution may negatively affect them in the future. This work formalizes the POUM hypothesis by explicitly modeling the voters' beliefs about their prospective incomes. Anticipation of future consumption creates an incentive for optimism and the poor will form overly optimistic beliefs and vote for low taxes if they value anticipation enough and if their optimism does not cause too drastic a change in tax policy. When beliefs are not conditioned on voting, the poor will always indulge in optimism and may even vote against their best interest. Furthermore, if the incomes of the rich increase as the incomes of the poor stagnate, the poor may demand less redistribution.

JEL classification: D31; D72; D83; D91
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1 Introduction

Why does it sometimes seem that the poor vote against their own self-interest? The prospect of upward mobility (POUM) hypothesis conjectures that the reason why the poor do not expropriate the rich and sometimes seem to vote against their self-interest is that they expect to move upward on the income ladder and fear that the higher redistribution may negatively affect their future consumption.

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The POUM hypothesis was introduced to the public choice literature by Bénabou and Ok (2001). They show that under favorable income dynamics, it is possible that more than half of the voters rationally expect above average future income. As a result, more than half of the voters prefer low redistribution and vote accordingly. While, according to the empirical evidence, both perceived (Ravallion & Lokshin, 2000; Rainer & Siedler, 2008; Cojocaru, 2014) and actual upward mobility (Alesina & La Ferrara, 2005; Alesina & Giuliano, 2011; Checchi & Filippin, 2004; Bénabou & Ok, 2001) seem to decrease voters' demand for redistribution, it also seems that perceived mobility and actual mobility do not necessarily correlate (Fischer, 2009; Alesina, Glaeser, & Sacerdote, 2001; Gottschalk & Spolaore, 2002; Kaufman, 2009).

The puzzle then, and what the model in Bénabou and Ok (2001) fails to explain, is why prospects of upward mobility decrease the demand for redistribution even in the absence of actual upward mobility. For instance, in the US, the perceived upward mobility is higher than in Europe, producing a higher POUM effect while there does not seem to be much difference in actual upward mobility across the Atlantic (Alesina et al., 2001; Gottschalk & Spolaore, 2002; Checchi, Ichino, & Rustichini, 1999; Lefranc & Trannoy, 2005; Björklund & Jäntti, 1997; Couch & Dunn, 1997). In addition, as noted by Alesina and Giuliano (2011) and Minozzi (2013), the assumptions underlying the model of Bénabou and Ok (2001) are restrictive and empirically implausible. Therefore, Alesina and Giuliano (2011) suggest that a more plausible mechanism for the POUM effect is overoptimism. I formalize the POUM hypothesis by explicitly modeling the voters' motivated beliefs about their prospective incomes and examine why and when these beliefs induce support for low redistribution.

A vast literature in experimental psychology on overconfidence supports Alesina and Giuliano's (2011) suggestion. Weinstein (1980) records the unrealistic optimism people have about their future life events: for instance, students tend to overestimate the likelihood of positive events such as getting a good job offer, earning a high salary and making good housing investments. In contrast, they underestimate the likelihood of health problems, unemployment and other negative events. Seaward and Kemp (2000) report how students overestimate their future incomes and job prospects and underestimate the time they need to pay off their student loans. This better-than-average-effect (Alicke & Govorun, 2005) is also not restricted to students and life events as demonstrated by Cross (1977) who find that 94% university faculty members assess their teaching ability above average. The inflated self-assessments are also linked to well-being (Taylor & Brown, 1988), and it is suggested that it is the desirability of certain beliefs that lead to biases (Kunda, 1990).

This paper models overoptimism in the context of voting for redistribution. In the model, voters collectively choose a linear tax rate under uncertainty over their future incomes. After voting, and before the realization, redistribution and consumption of their incomes, they anticipate their future consumption. This anticipation incentives optimistic expectations. Given the possibility of belief distortion, the motivated prospects of upward mobility emerge endogenously as a result of voters' optimal trade-off between anticipation and consumption. In addition, the model demonstrates how the collective action problem among the poor where the cost of being informed is the foregone gain in anticipation may lock the low-income voters in a bad equilirium where they vote against their best interest.

Furthermore, when the incomes of the rich increase, the perceived set of potential economic outcomes widen and anticipation of high outcomes becomes more attractive. Hence, an increase in income inequality may decrease the demand for redistribution. This prediction may help in understanding the observed stagnation and decline of demand for redistribution in the time of increasing inequality (Georgiadis & Manning, 2012; Rehm, Hacker, & Schlesinger, 2012; Korpi & Palme, 2003). A crucial component in the shift of income distributions has been the disproportional increases in the incomes of the rich (Piketty & Saez, 2014), the kind of change in income distribution that drives the decreases in the demand for redistribution in the model of this paper.

The paper proceeds as follows. In section 2, I briefly position this work into the literature. Section 3 presents the model and derives the conditions for the POUM effect. Section **??** extends the analysis of the model by studying the effects of changes in the underlying income distribution, presents welfare analysis and considers the case of nonstrategic belief formation and voting. Section 6 concludes. All proofs of the lemmas and propositions are collected in the appendix.

2 Related Literature

If the rational choice model with narrowly defined utility together with the Median Voter Theorem cannot be corroborated by empirical observations, at least one of these underlying assumptions, rational choice or median voter's power, must be the culprit. It might be the case that modeling voters as income maximizers does not capture all the relevant aspects of their decision-making or that the outcome that the electoral system provides does not reflect the preferences of the median voter.

In this work, the policy outcome is assumed to be the median voter's bliss point and the focus is on the former of these possible caveats.[1] Hence, this work continues the literature initiated by Romer (1975) and Meltzer and Richard (1981), which aims to explain the extent of redistribution in democratic economies by studying what determines the voters' demand for redistributive policies. The question asked is, how does

[1]Reasons for the latter could be, for instance, unequal political participation (Bénabou, 2000; Mahler, 2008), the political influence of the rich (Gilens, 2005), campaign contributions (Karabarbounis, 2011), economic inequality (Lupu & Pontusson, 2011; Solt, 2008), electoral systems (Iversen & Soskice, 2006; Cukierman & Spiegel, 2003; Austen-Smith, 2000), and interest groups (Dixit & Londregan, 1998).

the median voter decide on her vote.[2]

One line of literature focuses on mobility. First, income mobility, broadly speaking, refers to both upward and downward mobility. The premise is that instead of or in addition to current income, policy preferences depend on future income. When voters are worried that their incomes might decrease relative to others, they could use redistribution as an insurance against downward mobility. This would increase the demand for redistribution. The POUM, on the other hand, focuses on the possibility of upward mobility, having the opposite effect: When voters expect their incomes to increase relative to others, they oppose redistribution.

However, income mobility is also often connected to the roles of chance, circumstances, and effort in determining income. If voters perceive that the effort one exerts determines one's prospects, they believe in a mobile society, but if they believe that the circumstances have a major role in determining one's prospects, they believe in immobile society. Piketty (1995) studies how the interaction of income mobility and beliefs about the determinants of income affects voting. In the present work, incomes are exogenous and, in the spirit of the POUM hypothesis, beliefs about social mobility refer solely to beliefs about the levels of future incomes.

The first characterization of the POUM hypothesis is perhaps Hirschman's (1973) *tunnel effect* which conjectures that people's demand for redistribution decreases when they see the incomes of relatable people in their environment increase. This increases the expectations of their own incomes and they, therefore, tolerate more inequality. Hirschman and Rothschild (1973) are, however, agnostic to how these income expectations are formed.

The first formalization of the POUM hypothesis was provided by Bénabou and Ok (2001). Their approach is to maintain rational expectations and show that favorable income dynamics can allow more than half of the voters to expect above-average incomes. The voters vote for a redistribution policy, which will be in place for a predetermined time, and expect their incomes to evolve according to a stochastic transition function. The deterministic part of this transition function is concave, which allows the majority

[2]The starting point usually is the voter's current income, but preferences so narrowly defined have been unsatisfactory in explaining real-world tax policies (Bénabou, 1996; Borck, 2007; Luebker, 2014). Other factors explaining the demand for redistribution proposed in the literature are, for instance, efficiency costs of taxation (Meltzer & Richard, 1981), different individual (Piketty, 1995) and cultural (Corneo & Grüner, 2002; Alesina & Glaeser, 2004) histories and experiences, social preferences, such as altruism, inequality aversion and fairness considerations (Alesina & Angeletos, 2005; Alesina, Cozzi, & Mantovan, 2012; Alesina & Glaeser, 2004; Fong, 2001), status concerns (Gallice & Grillo, 2018), structure and organization of the family (Todd, 1985; Esping-Andersen, 1999; Alesina & Giuliano, 2010), and social mobility (Piketty, 1995; Hirschman & Rothschild, 1973; Bénabou & Ok, 2001). (See Alesina and Giuliano (2011) for a review on preferences for redistribution.) In addition to widening the scope of preferences, the literature has also studied the role of beliefs (Piketty, 1995; Alesina & Angeletos, 2005) and biased beliefs (Minozzi, 2013; Bénabou & Tirole, 2006; Bénabou, 2008). Given this rich set of explanations for the extent of redistribution, a parsimonious model seems unlikely, and a single factor should be interpreted as a part of the story, complementing and rivaling the other explanations.

of voters to believe that they will receive an above average income in the future. The stochastic part consists of skewed income shocks, which ensure that the original income distribution is preserved. This combination of skewed shocks and concave prospects lets the expected incomes and realized incomes diverge and makes the POUM effect possible with rational expectations. However, as discussed in Section 1, this approach is criticized.

Minozzi (2013) explains the POUM effect by abandoning rational expectations and letting voters form overly optimistic prospects about their future income. His model relies on a game theoretic multi-self approach, where each citizen has, without their knowledge, an "agent" who controls their beliefs and optimizes the trade-off between optimistic beliefs and material payoffs. Citizens receive utility from anticipation in period 1, and utility from consumption in period 2, when they receive their stochastic and exogenous incomes. The voter's objective function for belief formation consists of these two sources of utility. In choosing the optimal beliefs by solving the trade-off between anticipatory and outcome utility, the voter knows the prior prospects of the citizen and how the tax policy is dependent on the chosen beliefs. If the poor citizens value anticipation enough, they will end up with optimistic beliefs and vote for low redistribution.

I depart from Minozzi's (2013) most notably in the technology that voters use to distort their beliefs. The weakness of the model in Minozzi (2013) is in its naive non-Bayesian technology of belief distortion, which allows citizens to effectively decide what to believe and leaves them with no doubts of whether their recollections truly represent the reality. This might be too simplistic an assumption and an assumption that potentially misses important mechanisms of belief distortion as argued by Bénabou and Tirole (2002). The cognitive technology for belief distortion in the model is adopted from Bénabou and Tirole (2002, 2006) and generalized such that we are able to analyze a whole continuum of cognitive technologies varying in the constraints they impose on belief distortion allowing full and partial Bayesian rationality but also nesting the non-Bayesian technology used by Minozzi (2013). The conditions for the POUM effect are derived for each of these cognitive technologies, and it is shown that for a set of cognitive technologies the poor prefer optimism and low taxes over realism and high taxes. Also, it is demonstrated how the results of Minozzi's (2013) model are not robust to fully Bayesian updating of beliefs. Furthermore, in addition to strategic belief formation and voting, the only case considered by Minozzi (2013), here the case of sincere non-strategic belief formation and voting is studied and it is shown how voters who do not think that their beliefs and voting have a significant effect on the tax policy, always indulge in optimism and end up making nonoptimal decisions for themselves. Furthermore, I amend Minozzi's analysis by welfare results.[3]

[3]The POUM effect also appears in the model of Bénabou and Tirole (2006). In their model, voters are overly optimistic about their productive ability and, hence, future income. When they believe

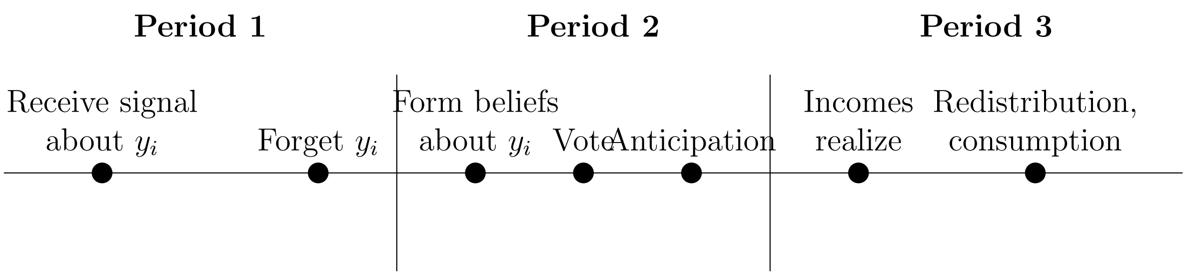

Figure 1: Timeline

3 The Model

There is a unitary continuum $i \in [0, 1]$ of voters who collectively decide on an income tax policy under uncertainty about their future incomes. In period 0, voters receive a signal conveying information about their prospective future incomes. In period 0, they also engage in various conscious and unconscious psychological processes of belief distortion, reality denial, and information avoidance which determine the signal they will remember in period 1. In the beginning of period 1, voters recall a signal and form beliefs about their future incomes based on their recollection. Then they vote for redistribution. They get to know the policy outcome immediately after the vote, and in the rest of period 1, they experience anticipatory utility as they anticipate their consumption which occurs in period 2, right after the incomes have been realized and redistributed. Figure 1 depicts the timeline.

3.1 Information and Beliefs

In period 0, each voter receives a signal $\sigma_i \in \mathcal{F} = \{F_L, F_H\}$. These signals are identical and independent. With probability p the signal is $\sigma = F_H$ and with probability $1 - p$ the signal is $\sigma = F_L$. F_H and F_L are cumulative distributions over the future income levels such that $\int_y y dF_H(y) > \int_y y dF_L(y)$ and $y \geq 0$.[4] Using the language of Minozzi (2013), we call the voters who receive signal $\sigma = F_H$ the likely rich and the voters who receive signal $\sigma = F_L$ the likely poor. With a large number of voters, a fraction p of

themselves to be abler than others, they prefer less redistribution. Although their model, as the present work, derives the POUM effect by letting voters optimize over their beliefs, their work differs from the current one in its mechanism for belief distortion. Specifically, what incentivizes the voters to hold biased beliefs differs. In their work, voters suffer from deficient willpower and form overly optimistic beliefs about their abilities in order to motivate themselves, and in this way to compensate for the imperfect willpower. That is, belief distortion works as a commitment device. In the current work, on the other hand, the beliefs are distorted because beliefs can be consumed and overly optimistic beliefs bring high anticipatory utility. However, these different incentives are not mutually exclusive, and probably both are at work.

[4]The signals are independent without loss of generality and to induce heterogeneity in the resulting income distribution. In general, the signals may be correlated. Perfectly correlated signals are natural when the unknown variable is common to voters in the sense that it reflects some general workings of the economy, as return to effort in Bénabou and Tirole (2006), government efficiency in Bénabou (2008) or expected value of a joint project in Bénabou (2013).

the population is likely rich and a fraction $1-p$ likely poor. Furthermore, we assume that the likely poor voters are a majority, that is, $p < \frac{1}{2}$. As voters are risk-neutral, a sufficient statistics for the analysis are the means of the distributions F_H and F_L: $y_H = \int_y y dF_H(y)$ and $y_L = \int_y y dF_L(y)$, the incomes that the likely rich and the likely poor, respectively, can expect to earn in period 2. In the following, we refer to these distributions by their means and let the signal set be $\{y_L, y_H\}$. The model is agnostic to the reasons of differences in incomes across voters and so we can generally think of them resulting from differences in (the evolution) of productivity or income relevant life-events. By having uncertainty with respect to future incomes, the model remains agnostic to whether income expectations are result of beliefs over ability or life events (or over the deterministic or stochastic term of the prodution function that generates income). Denote $\Delta y \coloneqq y_H - y_L$

After receiving a signal, each voter encodes a signal to be recalled in period 1.[5] As we will see, a likely poor voter has an incentive not to recall her true prospects. On the other hand, we make a sensible assumption that the likely rich voters will always choose to remember the signal they received and they, therefore, have no interesting decision to analyze. After all, if they underestimate their income, they lose anticipatory utility.[6] Hence, we focus mainly on the behavior of the likely poor voters. Formally, in period 0, a likely poor voter i chooses a recall rate $\lambda_i \in [0, 1]$ defined as $\lambda_i \coloneqq Pr[\hat{\sigma}_i = y_L | \sigma_i = y_L]$, where $\hat{\sigma}_i \in \{y_L, y_H\}$ denotes both the action she chooses in period 0 and the signal voter i recalls in period 1. The memory of voters is probabilistic and their actions in period 0 determine the probability of each recollection. We thus study type-symmetric equilibria where the likely rich always recall y_H and the likely poor recall y_L only with probability λ.

At the time of voting the voters selectively recall their income prospects and voter i's information is based on a recalled signal $\hat{\sigma}_i$. With probability λ_i, a likely poor voter will correctly recall $\hat{\sigma}_i = y_L$ and with probability $1 - \lambda_i$, she will recall $\hat{\sigma}_i = y_H$. The likely rich voters always recall $\hat{\sigma}_i = y_H$. The interpretation and encoding of information in period 0 and selective recall in period 1 are interpreted as all sorts of unconscious and conscious processes and actions that influence the availability of certain recollections. In equilibrium, voters act as if they were choosing optimal recall rates.

Voters may know that they have a tendency to forget bad news and remember good news and thus not fully trust their recollections. If voter i recalls $\hat{\sigma}_i = y_H$ in the second

[5]Voters have imperfect recall in the sense that they forget information. The underlying game theoretical construct to model this inconsistency is to model voters consisting of two players, their two temporal selves (see Bénabou and Tirole (2002)). Also, the parallel interpretation throughout the paper is that the parents have influence over what their offsprings belief when the offsprings are making voting decisions.

[6]This seems a very plausible conjecture but technically this is not that simple. Depending on the off-equilibrium path beliefs, an voter sending a low signal might end up with higher beliefs than when sending a high signal. In the appendix, we make an assumption about these off-equilibrium path beliefs to exclude this theoretical possibility.

period, she will assign a reliability $r(\lambda_i)$ to this signal:

$$r(\lambda_i) = Pr[\sigma_i = y_H | \hat{\sigma}_i = y_H] = \frac{p}{p + \chi(1-p)(1-\lambda_i)}, \tag{1}$$

where λ_i is given by the period-0 strategy of voter i. χ is the naivete parameter measuring the degree of Bayesian sophistication. $\chi = 1$ corresponds to full Bayesian rationality. In the other extreme, $\chi = 0$, and the reliability of received signal is always 1. This means that in period 1, voters will completely trust their recollections and that in period 0, they are completely in control of their period-1 beliefs. The role of χ will be analyzed extensively later. Only the likely poor send a signal $\hat{\sigma} = y_L$, so the reliability of low-income signal is always 1.

With probability $1 - \lambda_i$, a likely poor voter recalls $\hat{\sigma}_i = y_H$ and is an *optimist*. In period 1, an optimist expects a gross income

$$\hat{y}_i(y_H) = r(\lambda_i) y_H + (1 - r(\lambda_i)) y_L, \tag{2}$$

Note how a decrease in λ_i increases the probability of being an optimist and, as we will see, the expected anticipatory utility. However, the effect is nonlinear for $\chi > 0$ since the reliability decreases as λ_i increases. The more likely it is that a likely poor voter i memorizes a false signal, the less reliable signal $\hat{\sigma}_i = y_H$ becomes. The more voters try to distort their beliefs, the more cautious they are when they are forming their beliefs. This credibility of memories effectively constraints the optimism of voters.

With probability λ_i, a likely poor voter recalls $\hat{\sigma}_i = y_L$ and is a *realist*. Since the likely rich never recall low-income signals, in period 1, a realist expects a gross income

$$\hat{y}_i(y_L) = y_L. \tag{3}$$

The likely rich will recall $\hat{\sigma}_i = y_H$, and as they also do not know whether they truly are likely rich or likely poor, their expected income will coincide with the expected income of optimistic likely poor.

3.2 Preferences

In period 2, voters receive their incomes, pay taxes, and consume their disposable income. The government's budget is balanced, and all tax revenue collected via a linear income tax is transferred in equal lump-sums to voters. Voters derive utility linearly from their consumption:[7]

$$u_2(y_i, \tau) = (1 - \tau)y_i + \tau\bar{y}, \tag{4}$$

where τ is the income tax rate and $\bar{y} = qy_H + (1 - q)y_L$ is the mean income.

In period 1, voters form expectations about their future income and experience a utility flow due to anticipation. The preferences from the perspective of period 1 are given by

$$u_1(\hat{y}_i, \tau) = (s + \delta)[(1 - \tau)\hat{y}_i + \tau\bar{y}], \tag{5}$$

where the expectations are conditioned on the period-1 information, $\delta \in [0, 1]$ weights the consumption utility and $s \geq 0$ is the "savoring" parameter measuring the value of anticipation. The anticipatory utility is proportional to voter's expectations: The higher expectations she has, the higher is the utility flow. This gives voters an incentive to distort their beliefs. Setting $s = 0$ yields the standard case with no anticipatory utility and, thus, no incentive to distort beliefs. The discount factor and the savoring parameter are common to all voters.

The intertemporal utility from the period-0 perspective is

$$u_0(y_i, \hat{y}_i, \tau) = s[(1 - \tau)\hat{y}_i + \tau\bar{y}] + \delta[(1 - \tau)y_i + \tau\bar{y}]. \tag{6}$$

The expected period-1 flow utility depends on the information in period 1 and the expected period-2 flow utility depends on the information in the period 0. In period 0, voters know the true objective expectation of their incomes in period 2, but they also know that with upward biased beliefs they will receive higher utility in the period 1. Voters gain more utility if they have high hopes, but as we will see, with high hopes they will vote for low taxation, which then lowers their consumption in the last period.

[7] Risk aversion would induce a demand for redistribution in the form of insurance against downward mobility. This effect counters the effect of POUM. To not confound these effects, risk neutrality is assumed.

3.3 The Polity and Voting Decisions

The electorate votes for tax rate $\tau \in [\underline{\tau}, \overline{\tau}]$, $0 \leq \underline{\tau} < \overline{\tau} \leq 1$, in period 1. Voting follows the policy preferences (5). The preferred tax rates are[8]

$$\tau_i^* = \begin{cases} \underline{\tau} & \text{if} \quad \hat{y}_i \geq \bar{y} \\ \overline{\tau} & \text{if} \quad \hat{y}_i < \bar{y} \end{cases}. \tag{7}$$

If an voter expects in period 1 to earn an above average income in the period 2, she will vote for the low redistribution; if she expects to earn a below average income, she will vote for the high redistribution.

As the policy preferences given by (5) are single-peaked, the Median Voter Theorem (Black, 1948; Downs, 1957) applies and the chosen tax policy will be the tax rate preferred by the median voter. With two policy platforms, the median voter's opinion is the opinion of the majority. As we focus on type-symmetric decisions within the two groups of voters all of the likely rich choose $\hat{\sigma} = y_H$ and all of the likely poor choose the same λ. An optimist will always vote for $\tau = \underline{\tau}$ as seen from (7) and (2) and noting that $r(\lambda) \geq p$ for all $\lambda \in [0, 1]$. A realist will always vote for $\tau = \overline{\tau}$ by (3). Also, the likely rich will always vote for $\tau = \underline{\tau}$, similarly to the the optimistic likely poor. The policy outcome can be derived as a function of λ: The total share of voters expecting above average income is $p + (1-p)(1-\lambda)$ and the policy outcome τ^* depends on whether this share exceeds $\frac{1}{2}$ or not:[9]

$$\tau^* = \begin{cases} \underline{\tau} & \text{if} \quad \lambda < \frac{1}{2(1-p)} \\ \overline{\tau} & \text{if} \quad \lambda \geq \frac{1}{2(1-p)} \end{cases}. \tag{8}$$

3.4 Optimism

We now turn to the likely poor's motivated memory in period 0. Due to the discontinuity of the policy outcome in λ, the likely poor effectively have only two options to choose from. They either form optimal beliefs among those beliefs that support high taxation or optimal beliefs among those beliefs that support low taxation. Let $\overline{\lambda}$ be the optimal recall rate given $\lambda \geq \frac{1}{2(1-p)}$ and $\underline{\lambda}$ the optimal recall rate given $\lambda < \frac{1}{2(1-p)}$. The high recall rate $\overline{\lambda}$ supports low taxation; the low recall rate $\underline{\lambda}$ supports high taxation. For

[8] We assume that an indifferent voter votes for low taxes. This assumption determines the tax policy in the low tax equilibrium of the model in the case of $\chi = 1$. Alternatively we could suppose, that there is an arbitrarily small amount of wastage involved in taxation, or that the voters deviate an arbitrarily small amount from the full Bayesian rationality. Both of these would make the voter with mean income strictly prefer low taxation.

[9] The assumption that the policy outcome is $\overline{\tau}$ in case of $\lambda = \frac{1}{2(1-p)}$ ensures that an optimal choice of λ exists for all $s > 0$.

recall rate $\lambda \in \{\underline{\lambda}, \overline{\lambda}\}$ and tax rate $\tau \in \{\underline{\tau}, \overline{\tau}\}$, the expected payoff is

$$\begin{aligned} U_0(\lambda, \tau) &= \lambda u_0(y_L, y_L, \tau) + (1-\lambda) u_0(y_L, y_H, \tau) \\ &= (1-\tau)\big[s\iota_{gross}(\lambda) + \delta y_L\big] + (s+\delta)\tau\bar{y}, \end{aligned} \tag{9}$$

where

$$\iota_{gross}(\lambda) := (1-\lambda)\hat{y}_i(y_H) + \lambda y_L \tag{10}$$

is the expectation of expected gross income of a likely poor voter in period 1 from the point of view of period 0 as a function of λ. With probability $\underline{\lambda}$, a likely poor voter recalls $\hat{\sigma}_i = y_L$ and forms realistic beliefs. With probability $1 - \underline{\lambda}$, a likely poor voter recalls $\hat{\sigma}_i = y_H$ and forms optimistic beliefs weighted by the reliability of the signal. In both cases, the likely poor's period-2 consumption is low.

The utility difference between these two choices, or the *incentive to optimism* is:

$$U_0(\underline{\lambda}, \underline{\tau}) - U_0(\overline{\lambda}, \overline{\tau}) = s[\iota_{net}(\underline{\lambda}) - \iota_{net}(\overline{\lambda})] + \delta(\overline{\tau} - \underline{\tau})p\Delta y, \tag{11}$$

where

$$\iota_{net}(\lambda, \tau) := (1-\tau)[(1-\lambda)\hat{y}_i(y_H) + \lambda y_L] + \tau\bar{y} \tag{12}$$

is the expectation of the expected consumption, i.e., up to a positive factor, the expected anticipation in period 1 given the choice of λ and tax policy τ. The first term in (11) captures the gain in anticipation and the second term the loss in net income. Whether the likely poor have high or low recall rate depends on whether the incentive to optimism is positive and this depends on the value of anticipation.

Lemma 1. *The likely poor choose $\lambda = \underline{\lambda} = 0$ if*

$$s \geq s^* := \frac{\delta(\overline{\tau} - \underline{\tau})p\Delta y}{\iota_{net}(\underline{\lambda}, \overline{\tau}) - \iota_{net}(\overline{\lambda}, \underline{\tau})}. \tag{13}$$

Otherwise they choose $\lambda = \overline{\lambda} = \frac{1}{2(1-q)}$.

The nominator of (13) represents the difference in period-2 consumptions in the two different tax regimes. Clearly, when $\overline{\tau}$ decreases or $\underline{\tau}$ increases, this difference becomes smaller. When this difference becomes smaller the loss in the period-2 consumption when choosing $\lambda = \underline{\lambda}$ over $\lambda = \overline{\lambda}$ decreases. If the nominator decreases, s^* decreases proportionally and the POUM effect becomes more likely. The denominator of (13) is proportional to the difference in expected anticipatory utility of the likely poor between their choices of low or high recall rate. When this difference increases, the likely poor have more to gain in anticipation and belief distortion becomes more attractive. If the denominator increases, s^* decreases and the POUM effect becomes more likely.

3.5 Equilibrium

We have defined s^* to be a threshold such that if $s \geq s^*$, then voters value anticipation enough for the gain in anticipatory utility to outweigh the loss of income, and the likely poor will be optimistic enough to vote for a low tax rate. If, on the other hand, $s < s^*$, then the anticipation is not enough to compensate for the lost income and the likely poor will remain realistic enough to vote for a high tax rate.

Proposition 1. *An equilibrium is a 4-tuple* $(y_H, \lambda^*, r(\lambda^*|\chi), \tau^*)$.[10]

(i) If $s \geq s^*$, there is an equilibrium in which the likely poor choose $\lambda^* = \underline{\lambda} = 0$, the likely rich choose $\hat{\sigma} = y_H$, and the policy outcome is $\tau^* = \underline{\tau}$.

(ii) If $s < s^*$, there is an equilibrium in which the likely poor choose $\lambda^* = \overline{\lambda} = \frac{1}{2(1-q)}$, the likely rich choose $\hat{\sigma} = y_H$, and the policy outcome is $\tau^* = \overline{\tau}$.

If voters could not manipulate their expectations or if they did not have any incentives to distort their beliefs, they would vote according to their objective prospects, and the unique equilibrium would be the likely poor voting for high taxes and the likely rich voting for low taxes. The median voter would be among the likely poor, and the unique policy equilibrium would set taxes high. The possibility of subjective beliefs that differ from the objective standard allow additional equilibria with other policy outcomes.

If voters value anticipatory utility enough the motivated prospects of upward mobility support low taxes. How much is enough depends on the threshold s^*. The higher s^* is, the less likely the POUM effect is, and conversely, the lower s^* is, the more likely we will observe low taxation. This threshold varies with the parameters of the model.

3.6 Constraints of Optimism

The model nests Minozzi's (2013) model of taxation and endogenous prospects of upward mobility. For parameter values $\delta = 1$, $\chi = 0$ $\underline{\tau} = 0$ and $\overline{\tau} = 1$ we have[11]

$$s^* = \frac{q}{1-q} = \frac{\bar{y} - y_L}{y_H - \bar{y}}. \tag{14}$$

[10]There is a third type of symmetric equilibrium, where all voters choose $\hat{\sigma} = y_H$ and the policy outcome is $\tau = \underline{\tau}$ even if $s < s^*$ as there would be no unilateral incentive to deviate.

[11]Minozzi's (2013) model which abstracts from discounting yields $\delta^* = \frac{n-m}{m} = \frac{\bar{y}-y_p}{y_r-\bar{y}}$, where δ^* is the threshold of the savoring parameter, n is the (finite) number of voters, m is the (finite) number of the likely poor, y_p is the income of the likely poor, y_r is the income of the likely rich and $\bar{y}$ is the mean income.

On the other hand, keeping $\underline{\tau} = 0$ and $\overline{\tau} = 1$ but allowing voters' inference approach Bayesian rationality, we have

$$\lim_{\chi \to 1} s^*(\chi) = \infty. \tag{15}$$

The threshold required for the prospects of upward mobility to yield a low-tax equilibrium approaches infinity as the inference of voters approaches full Bayesian rationality. This means that with full Bayesian rationality the importance of anticipation s can never be above s^* and it can never be optimal for the likely poor to form beliefs that support low taxation. That is, on contrary to the special case of Minozzi's (2013) model, where $\chi = 0$, if we acknowledge that the people cannot simply choose their beliefs and let $\chi > 0$, the threshold s^* increases dramatically in χ and in the extreme case of full Bayesian rationality, the POUM effect can never occur.

To see why fully Bayesian likely poor voters can never be better off with low taxes, consider again the incentive to optimism given in (11). Plugging in the optimal recall rate $\underline{\lambda} = 0$, the incentive to optimism can be written as

$$U_0(\underline{\lambda}, \overline{\tau}) - U_0(\overline{\lambda}, \underline{\tau}) = -\delta(\bar{y} - y_L) + s[r(0|\chi) - p]\Delta y, \tag{16}$$

The second term in the right hand side is the gain in anticipation if an voter chooses $\underline{\lambda}$ over $\overline{\lambda}$. Noting that $r(0|\chi) \to 1$ as $\chi \to 0$ and $r(0|\chi) \to q$ as $\chi \to 1$, it is easy to see how the value of the second term goes to zero as $\chi \to 1$ and why it does not when $\chi = 0$. The incentive to optimism is at its maximum when $\chi = 0$ and as voters' inference approaches full Bayesian rationality the utility gain from anticipation vanishes.

The reliability which the voters use to weight the information of their recollection plays a crucial role here. For $\chi = 1$, the reliability r is an increasing function of λ. The more realistic the likely poor are, the more reliable signal $\hat{\sigma}_i = y_H$ is. On the other hand, when the likely poor systematically memorize and recall $\hat{\sigma}_i = y_H$, they know that no matter what is their true signal, they recall $\hat{\sigma}_i = y_H$. In this case, the signal does not carry any information, and voters' beliefs correspond to the prior distribution, $r(0|\chi) = q$. However, when the degree of Bayesian sophistication decreases, the reliability becomes less dependent on λ, and the optimistic poor put more weight on their pleasant recollection. When $\chi = 0$, the reliability is independent of λ and no matter how optimistic the likely poor are, they always fully trust their recollections.

It is instructive to see how the period-0 expectation of expected period-2 income in period 1, and expected anticipatory utility which is proportional to the expected income, varies with λ and χ.

This function is plotted in Figure 2 for different values of χ. The lowest curve corresponds to the case $\chi = 1$. As voters put more and more weight on signal $\hat{\sigma}_i = y_H$ in their period-0 strategy, that is, as they become more and more likely to remember

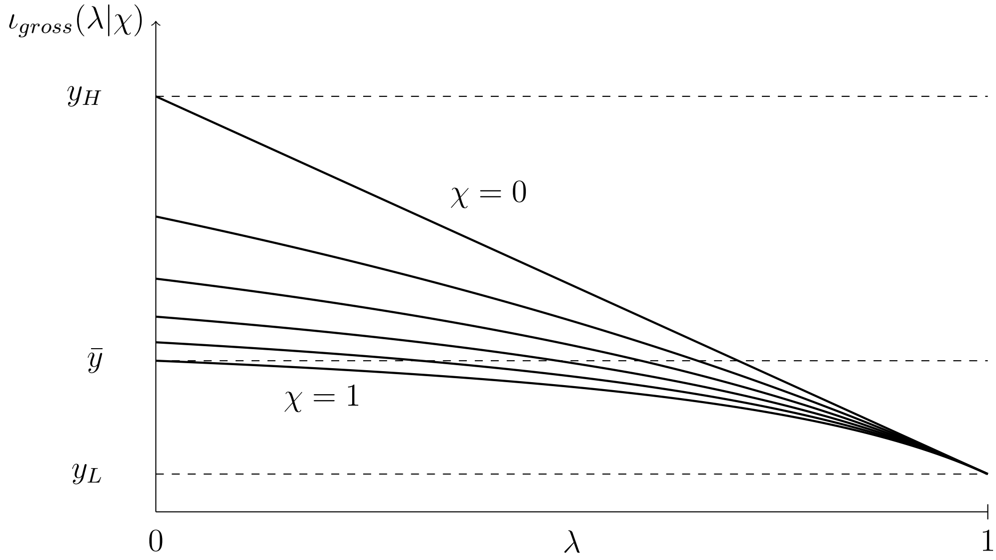

Figure 2: $\iota_{gross}(\lambda|\chi)$ for $\chi \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$

$\hat{\sigma}_i = y_H$, the expected income approaches the average income. In the case of $\lambda = 0$, each of the likely poor and each of the likely rich always recall signal $\hat{\sigma}_i = y_H$. As everyone is pooling on the same signal, receiving this signal does not give any information, and voters rely on the prior information when assessing their future income. In the case of full Bayesian rationality, it is, therefore, not possible for voters to achieve above average expectations. As they expect average income in the fully expropriating high tax regime, they cannot possibly improve their utility by distorting their beliefs and voting for low taxes.

On the contrary, when $\chi < 1$, voters can achieve above average expectations, and they, therefore, can have a gain in anticipatory utility to trade off against the lost income in the low tax regime. For voters with $\chi < 1$, a decrease in λ does not affect the reliability of the signal as much as it affects for the Bayesian rational voters. In the limiting case of $\chi = 0$, represented by the linear curve in Figure 2, the reliability is independent of λ, and all voters can believe to be of type y_H. The expectations of naive voters are not as constrained as the expectations of Bayesian voters. The naive voters can, therefore, achieve higher hopes and higher anticipatory utility than their Bayesian counterparts.

In the case of full Bayesian rationality, the wishful beliefs of the likely poor are bounded above to the average income, which is what they expect to receive if $\tau^* = 1$ as well. They cannot, therefore, increase their anticipatory utility by distorting their beliefs. However, what if they could not expect the incomes to be fully equalized under the high tax policy. Then they might be able to increase their anticipatory utility by distorting their beliefs even if they still ended up with expectations of average income. The case of $\overline{\tau} = 1$ and $\underline{\tau} = 0$ is too unrealistic a simplification and we therefore turn now to the more general case of $\tau \in [\underline{\tau}, \overline{\tau}]$.

Proposition 2. *The threshold s^* increases in χ.*

Even if the POUM effect is now possible for all $\chi \in [0,1]$, it can still be questioned whether it is feasible for all $\chi \in [0,1]$. Again, the voters may have to value anticipation more than consumption to prefer low taxes if the range of the feasible tax rates is big enough. To see this, consider the threshold value s^* when $\chi = 1$:

$$s^*(1) = \delta \frac{(\overline{\tau} - \underline{\tau})(1-q)}{(1-\overline{\tau})q}. \tag{17}$$

Now $s^*(1) > \delta$, for all pairs $(\underline{\tau}, \overline{\tau})$, such that $\overline{\tau} > (1-q)\underline{\tau} + q$. We could argue that within a jurisdiction, the range of feasible tax rates is small and, hence, the POUM effect is feasible also for a sophisticated cognitive technology. On the other hand, as discussed, fully Bayesian sophistication may not be the correct specification in the belief distortion technology to represent people's beliefs about their future incomes and their voting behavior.

4 Comparative Statics

As already seen, given the value of anticipation s, the threshold s^* determines whether the POUM support low-tax equilibrium. The comparative statics of s^*, therefore, reveal how the conditions for the low-tax equilibrium vary as the parameters of the model change.

The upper limit of the tax is relevant when the likely poor choose $\lambda = \overline{\lambda}$, since then the resulting policy is high taxes. By imposing a restriction on how much of the income can be redistributed we make the prospects of choosing $\lambda = \overline{\lambda}$ worse. Consider the effects on the period-2 consumption and period-1 anticipatory utility separately. First, a decrease in the upper limit of the tax rate decreases the period-2 consumption of the likely poor in the high tax regime, which makes voting for high taxes less rewarding. Second, for those of the likely poor who end up being realists, the lower consumption in period 2 implies lower anticipation in period 1. Those of the likely poor who end up being optimists will expect above-average incomes, and they will, therefore, gain in anticipatory utility as the upper limit of the tax decreases. However, the overall effect unambiguously decreases the attractiveness of choosing $\lambda = \overline{\lambda}$. Proposition 3 formalizes this total effect of the upper limit of the tax rate.

Consider next what happens when we set a positive lower limit for the allowed tax rate. The prospects of choosing $\lambda = \underline{\lambda}$, are now better. The likely poor choosing $\lambda = \underline{\lambda}$ leads to low taxes, so it is here where the lower limit of the tax rate is interesting. Again, there is an effect on the period-2 consumption and on the period-1 anticipation. First, even if the likely poor vote for low taxation, redistribution does not vanish altogether. Since they are trading their optimism against redistribution, the cost of optimism is now lower. The reduction in their period-2 consumption is not as drastic as with the possibility of complete laissez-faire. This makes choosing high anticipation and low taxes more attractive. Second, when choosing $\lambda = \underline{\lambda}$, all of the likely poor end up being optimists. If they then anticipate above average income, that is, if $\chi < 1$, then an increase in the lower limit of the tax rate will decrease their anticipatory utility. The less sophisticated the voters are, the more they expect to earn, and the higher is the decrease in their anticipation. The effect on anticipatory utility is opposite to the effect on consumption. The effect on consumption, however, dominates. Proposition 3 formalizes this.

Proposition 3. *The threshold s^* increases in $\overline{\tau}$ and decreases in $\underline{\tau}$.*

The utility from choosing $\lambda = \underline{\lambda}$ increases with $\underline{\tau}$ and the utility from choosing $\lambda = \overline{\lambda}$ decreases with $\overline{\tau}$. This means that the utility cap between choosing $\lambda = \underline{\lambda}$ and $\lambda = \overline{\lambda}$ increases as the range of allowed tax policies decreases. This utility cap is, by definition, the incentive to optimism. An increase in the incentive to optimism then leads to less stringent conditions for the POUM effect.

Following Minozzi's (2013) analysis, we first examine the changes in y_L and y_H holding the average income constant. Proposition 4 collects these results.

Proposition 4. *Holding the average income constant, the threshold s^* decreases in y_L and y_H.*

If the incomes of the likely rich increase such that the average income stays constant, the conditions for the POUM effect become looser. Similarly, if the incomes of the likely poor increase such that the average income stays constant, the conditions for the POUM effect become again looser. We insist on holding the average income constant because it makes the effects interesting. The average income $\bar{y}$ is a function of both y_L and y_H and taking this into account gives us $\frac{\partial s^*}{\partial y_H} = \frac{\partial s^*}{\partial y_L} = 0$ as can easily be seen by noting that s^* in (13) is independent of both y_H and y_L. So by letting the average income adapt to the changes in the incomes of the likely poor or the likely rich, the condition for the POUM effect would not change.

Looking at the incentive to optimism given in (11) gives us an idea of how to interpret Proposition 4. For the voters, changes in the average income imply changes in the transfers they receive, whereas changes in either y_L or y_H imply changes in the expectations of their pre-tax income. That is, holding average income constant means holding the tax revenue and transfers constant, whereas increases in the high and low levels of income mean increased expectations of gross income. Increased prospects of gross income, when the transfers are expected to stagnate, make optimism more rewarding.[12] This kind of change in the income distribution could occur, for instance, if the income tax is regressive such that the increase in the incomes of the likely rich

[12] For empirical evidence on beliefs responding to the sizes of potential prizes, see Coutts (2019).

leads to less than a proportional increase in the tax revenue. We could also interpret the income levels y_L and y_H more loosely as what the likely poor perceive these income levels to be. The perceived income of the likely rich could change without affecting the tax revenue, for instance, if the incomes in other jurisdictions change and the likely poor observe this or if the consumption habits of the likely rich change towards more conspicuous consumption.

Similarly, the change in the average income has no effect as such, $\frac{\partial s^*}{\partial \bar{y}} = 0$, but holding y_L and y_H constant and letting $\bar{y}$ change gives us

Proposition 5. *Holding y_L and y_H constant, the threshold s^* increases in $\bar{y}$.*

The case of holding y_L and y_H constant and letting $\bar{y}$ change mirrors the previous discussion. If the likely poor expect increased transfers but the prospects of gross income stay the same, then realism becomes more attractive.

The changes in the fraction of the likely rich produce more complicated effects mainly because the reliability and the optimal recall rate $\overline{\lambda}$ are functions of p. We only characterize the effects. Consider a change in the income distribution where the proportion of the likely rich becomes smaller. A decrease in p has three effects. First, it decreases the average income and the tax revenue and, therefore, makes realism less attractive. Second, it decreases $\overline{\lambda}$, the optimal choice if the likely poor opt for high taxes. When there are fewer likely rich voters voting for low taxes, it allows the likely poor to be more optimistic even if they opt for high redistribution. This makes realism more attractive. Third, as p and, hence, $\overline{\lambda}$ decrease, they both contribute to decreasing the reliability of the signal $\hat{\sigma}_i = y_H$ and, therefore, make the anticipated income lower and optimism less attractive.

All effects that work via the reliability of recalled signal depend crucially on χ. Hence, for low values of χ, the reliability does not depend that much on the prior distribution or $\overline{\lambda}$ and the first effect dominates. In this case, POUM effect becomes more likely as the prospects of choosing $\lambda = \overline{\lambda}$ are now worse. For high values of χ, the reliability is highly dependent on the prior and $\overline{\lambda}$ and the second and third effect dominate. In this case, a decrease in p makes POUM less likely. For intermediate values of χ, the relative dominance of these effects varies, and the total effect is nonmonotonic.

5 Welfare

When utilities are linear in consumption the aggregate utility is not sensitive to the distribution of consumption. Therefore, the aggregate utility is trivially maximized by maximizing anticipation and, thus, the aggregate utility as a measure of welfare is not meaningful here. This section, therefore, after a brief discussion on the distribution and aggregation of consumption and anticipation, focuses on the welfare of the likely poor and the likely rich separately.

By the additivity of consumption utility and anticipatory utility, the aggregate levels of these two components of utility can be studied separately. Furthermore, as the utilities with respect to consumption and anticipation are both linear, the welfare consists of aggregate consumption and aggregate anticipation.

Since redistribution does not produce any wastage here, the aggregate consumption stays constant at the average consumption throughout the analysis. Only the distribution of consumption between the likely poor and the likely rich varies depending on the chosen tax policy. The higher is the tax rate, the more equally the aggregate consumption is distributed among the likely rich and the likely poor.

The more novel component of welfare is the aggregate anticipation. This is the sum of anticipation of those voters who recalled $\hat{\sigma}_i = y_L$ and of those who recalled $\hat{\sigma}_i = y_H$. A fraction $(1-q)\lambda$ of voters recalls $\hat{\sigma}_i = y_L$ and they anticipate a gross income of y_L. A fraction $q + (1-q)(1-\lambda)$ of voters recalls $\hat{\sigma}_i = y_H$ and they anticipate a gross income of $r(\lambda)y_H + (1-r(\lambda))y_L$. Note especially that those who truly belong to the likely rich anticipate the same gross income as those of the likely poor who recall signal $\hat{\sigma}_i = y_H$. The aggregate anticipatory utility derived from the anticipation of gross income is[13]

$$(1-q)\lambda s y_L + [(1-q)(1-\lambda) + q] s \hat{y}_i(y_H). \tag{18}$$

The aggregate anticipation depends on the constraints of the cognitive technology and the awareness choices of the likely poor. For $\chi = 1$, the aggregate anticipatory utility is constant at $s\bar{y}$. Bayesian rationality imposes a constraint on beliefs such that on average, voters expect average income. Therefore, for the special case of $\chi = 1$, the aggregate anticipation is similar to the aggregate consumption in the sense that only the distribution of the anticipation varies. As the Bayesian constraint is relaxed and values of $\chi < 1$ are allowed, the aggregate anticipation can exceed the anticipation of average income, and it is no more independent of λ. In this case, the aggregate anticipation is maximized at $\lambda = 0$.

The counterintuitive consequence of the assessment of the reliability of recollections is that for all $\chi > 0$, the likely rich will underestimate their future income. If all of the likely poor choose to memorize the signal $\hat{\sigma}_i = y_H$, then all voters, the likely rich and the likely poor, will recall this signal in period 1. When the likely rich assess the reliabilities of their recollections, they know that no matter which signal they receive in period 0, they will always recall $\hat{\sigma}_i = y_H$. In the case of full Bayesian rationality, this means that the likely rich use their prior information to form their expectations and, hence, underestimate their future income.[14] If, on the other hand, the likely poor

[13]The aggregate anticipation of net income is

$$(1-\tau)\left[(1-q)\lambda s y_L + [(1-q)(1-\lambda) + q] s \hat{y}_i(y_H)\right] + s\tau\bar{y}$$

and it behaves similarly to the aggregate anticipation of gross income.

[14]Cruces, Perez-Truglia, and Tetaz (2013) find evidence, that in addition to the poor overestimating

choose to memorize the signal they received, then the likely rich, after recalling $\hat{\sigma}_i = y_H$ know that the only way to recall this signal is to be likely rich. In this case, they form accurate expectations.

This dependence of the anticipation of the rich on the awareness rates of the likely poor can be thought of as a negative externality. As λ decreases, the likely poor are more and more optimistic and the likely rich more and more pessimistic. That is, when the likely poor engage in optimism, they redistribute anticipation. If $\chi = 1$, and the likely poor choose $\lambda = 0$, they equalize all anticipation. In this case, the average anticipation is constant, and the gain in anticipatory utility of the likely poor is exactly offset by the loss in the anticipatory utility of the likely rich. The strength of this externality and its redistributive effect increase in χ. For completely naive voters, the reliability of recollection is independent of λ, and there is no externality.

This externality should, however, not be thought of as a causal relationship between the cognitive processes of different voters, but as an *externality across information states*, as Bénabou and Tirole (2002, p. 907) put it. The likely rich do not underestimate their prospects because the likely poor overestimate theirs, but because they know that had they themselves been likely poor, they might still have memorized the signal $\hat{\sigma}_i = y_H$. The negative externality for the likely rich is, therefore, caused by their own information processing strategy, that is, by their own hypothetical action in an alternative history.

If the likely poor choose the low tax equilibrium with high expectations, they are clearly better off in this equilibrium. The pessimism of the rich, however, raises the rather surprising question of whether the likely rich are worse or better off in the low tax equilibrium. In the standard case, without anticipatory utility, the rich are clearly better off in the low tax equilibrium. When we take the anticipation into the analysis, the rich still have higher period-2 consumption in the low tax equilibrium, but the negative externalities across information states erode their anticipation in period 1. We now see, which of these effects dominates.

In the low tax equilibrium, the utility of the likely rich from the viewpoint of period 0 is

$$u_{0,i}(y_H, y_H, \underline{\tau}) = s\,[(1-\underline{\tau})[r(\underline{\lambda})y_H + (1-r(\underline{\lambda})y_L] + \underline{\tau}\bar{y}] + \delta[(1-\underline{\tau})y_H + \underline{\tau}\bar{y}], \qquad (19)$$

their position in the income distribution, the rich tend to underestimate theirs. However, their proposed mechanism is different: voters estimate the overall income distribution by extrapolating from the incomes of their reference group. If the reference group does not well represent the overall income distribution, the estimates will be biased. Also, underconfidence is a well-documented phenomenon in the literature of psychology and tends to concern those with the best prospects. See, for instance, Moore and Healy (2008).

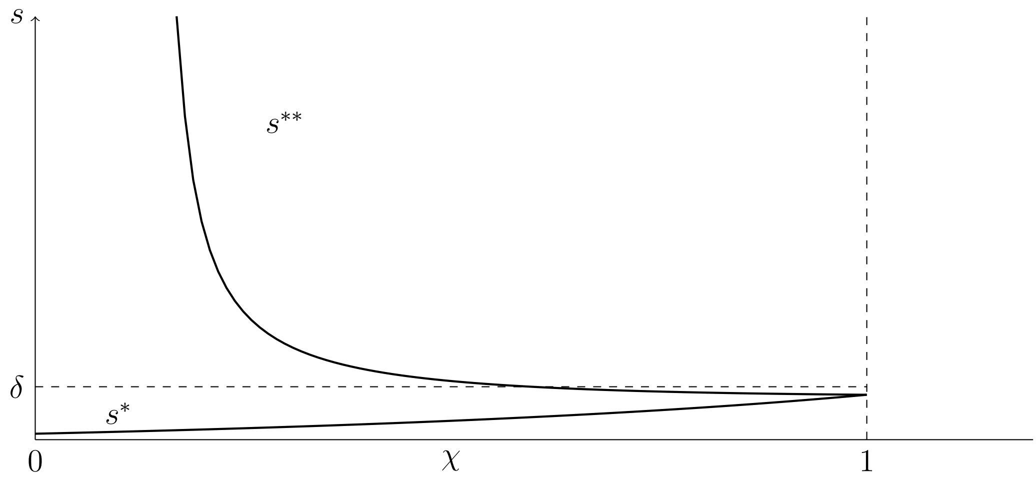

Figure 3: s^* and s^{**} as a function of χ

and in the high tax equilibrium the utility of the likely rich is

$$u_{0,i}(y_H, y_H, \overline{\tau}) = s\left[(1-\overline{\tau})[r(\overline{\lambda})y_H + (1-r(\overline{\lambda})y_L] + \overline{\tau}\bar{y}\right] + \delta[(1-\overline{\tau})y_H + \overline{\tau}\bar{y}]. \tag{20}$$

Again, whether the anticipation effect dominates depends on the value of anticipation. The likely poor choosing optimism and low taxes makes the likely rich worse off if (20) is greater than (19). If $(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q > 0$, the condition for this reads:

$$s < \frac{-\delta(\overline{\tau}-\underline{\tau})(1-q)}{(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q}. \tag{21}$$

Since the denominator is positive and the nominator negative, the right-hand side of (21) is negative. As $s \geq 0$, the condition is never satisfied, and the likely rich are always better off in the low tax equilibrium.[15] If, on the other hand, $(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q < 0$, the condition for the likely rich to be worse off in the low tax equilibrium reads:

$$s > \frac{\delta(\overline{\tau}-\underline{\tau})(1-q)}{(\overline{\tau}-\underline{\tau})q + (1-\overline{\tau})r(\overline{\lambda}) - (1-\underline{\tau})r(\underline{\lambda})} \equiv s^{**}(\chi). \tag{22}$$

Obviously, whether the likely rich are worse off in the low tax equilibrium is an interesting question only when the low tax equilibrium is possible. Figure 3 depicts s^* and s^{**} as a functions of χ. As we have seen, the low tax equilibrium occurs if $s \geq s^*$. By definition of s^{**}, the likely rich are worse off in the low tax equilibrium if $s > s^{**}$. For $\chi = 1$ the thresholds s^* and s^{**} coincide, and for $\chi \in [0, 1)$ s^{**} is greater than s^*.

[15] The condition is also never satisfied when $(1-\underline{\tau})r(\underline{\lambda}) - (1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q = 0$, since the right-hand side of (21) approaches infinity as the denominator approaches zero from the right and the condition then would require s to be greater than infinity. When the denominator approaches zero from the left, the right-hand side approaches negative infinity and the argument in the text applies.

Therefore, only for the fully Bayesian voters the optimism of the likely poor necessarily makes the rich worse off. For $\chi < 1$ this does not need to be the case.

Proposition 6. *Whether the likely rich are worse off in the low tax equilibrium depends on the degree of the Bayesian sophistication and the value of anticipation.*

(i) For $\chi = 1$, the likely rich are worse off in the low tax equilibrium than in the high tax equilibrium.

(ii) For $\chi < 1$, the likely rich are worse off in the low tax equilibrium only if $s > s^{**}(\chi)$.

In Figure 3, for parameter pairs (s, χ) below the lower curve, the POUM effect does not occur. Between the two curves, the POUM effect occurs and it makes the likely rich better off. Above the upper curve, the POUM effect occurs, and it makes the likely rich worse off.

Interestingly, an implication of the model is that the fully Bayesian likely rich are worse off with low taxes if the value of anticipation is high enough for likely poor to choose optimism and low taxes. Again, however, completely sophisticated cognitive technology might be of only theoretical interest. The threshold value s^{**} goes up fairly rapidly for $\chi < 1$, which makes this result less relevant.

5.1 Nonpivotal Voting

The beliefs are most likely to be distorted by desires if the individual cost of holding biased beliefs is small, as is the case in voting if the probability of being pivotal is negligible (Bénabou & Tirole, 2016; Caplan, 2011). Here we consider the case where voters do not consider themselves to be pivotal in the determination of the tax policy and, thus, form their beliefs without taking into account how it affects their policy preferences and voting.

In the model, voters trade their optimism against redistribution. If we let voters ignore this trade-off, the only the credibility constraints of belief manipulation contstrain optimism. Thus, taking τ^* as given, the dominant action for the likely poor is to choose $\lambda = 0$ for all $s > 0$: The lower λ they choose, the higher anticipatory utility they expect. The loss of income and consumption in period 2 due to less extensive redistribution does not enter the trade-off since the voters do not think they can influence the policy outcome. In the unique equilibrium, all voters recall $\hat{\sigma} = y_H$, they expect at least average income, and the tax policy is $\tau^* = \underline{\tau}$. This is the equilibrium even if the likely poor do not value anticipation much and are worse off than if they had all been realists and voted for high taxes.

Another way to motivate nonpivotal voting is to derive it as a limiting case of the model with strategic behavior. When the range of the feasible tax rates goes to zero, the threshold s^* goes to zero as well: $\overline{\tau} - \underline{\tau} = 0$ implies $s^*(\chi) = 0$, and choosing $\lambda = \underline{\lambda}$

over $\lambda = \overline{\lambda}$ is optimal for all $s > 0$. When the upper and lower bounds of the tax policy coincide, the likely poor cannot affect the tax rate by voting, and it is optimal for them to indulge in optimism.

When the likely poor do not think that their beliefs will influence the policy outcome, they maximize their utility by maximizing their optimism. In contrast to the case of strategic belief formation, which admittedly is a strong requirement on the behavior of the voters, sincere voting always leads to the POUM effect.

Proposition 7. *If the likely poor do not condition their belief and voting choices on the tax policy, then, for all $s > 0$, there is a unique equilibrium, where $\lambda^* = 0$, the likely rich recall $\hat{\sigma}_i = y_H$, and $\tau^* = \underline{\tau}$.*

Proposition 8. *If $s < s^*$, the likely poor are worse off in the low tax equilibrium, than if they had coordinated on voting for high taxes.*

A free-riding problem emerges among the likely poor: for each, it is individually rational to indulge in optimism, but with coordinated actions they could increase their payoffs. This is similar to the public goods game, where the individually rational voters do not contribute even if they would all be better off by contributing. Here the public good is the redistribution, and the cost of contribution is lower anticipatory utility.[16] This collective action problem is thus similar to the one discussed by Caplan (2000, 2001b, 2001a): Voters may indulge in biased beliefs when there is no cost for which. This is problematic for voting since, the low probability of pivotality keeps the cost of biased beliefs low while biased voters may vote for suboptimal policies. Relative to Caplan's work, here both the demand for belief biases and the trade-off between biases and material costs are explicitly modeled.

6 Conclusion

Overoptimism can provide a mechanism for the POUM hypothesis. This paper formalizes this by modeling the means and reasons for belief distortion and derives the conditions under which the poor majority of voters distort their beliefs enough to prefer low taxes in the time of voting. The poor do not expropriate the rich because they themselves overly optimistically believe to be rich someday, and they value these beliefs. Thus, motivated prospects of upward mobility emerge endogenously as a result of voters' choices between anticipation and consumption. The crucial factors in these choices are the value of anticipation and the relative differences in anticipation and consumption between the potential equilibria.

[16] However, the likely poor coordinating on realism to support high taxes is not a Pareto improvement when considering the whole electorate, as providing a public good in a public good game is. As seen in section 5, the likely rich are worse off in the high tax equilibrium if $s > s^{**}$ which is never the case when $s < s^*$.

First, the more the poor expect to gain in anticipation when forming biased beliefs, the more biased these beliefs will be. Specifically, if the incomes or perceived incomes of the rich increase while transfers stagnate, the poor will be more likely to indulge in optimism and vote for low taxes. Hence, the striking result is that contrary to the benchmark model of Meltzer and Richard (1981), where the increase in inequality always increases the demand for redistribution, in the model proposed here, an increase in inequality can decrease the demand for redistribution. The mechanism proposed may help understand why the demand for redistribution has not increased and may even have declined (Georgiadis & Manning, 2012) in the times of increasing income inequality driven especially by the top-income shares (Piketty & Saez, 2014).

Second, the less the likely poor expect to lose in consumption when forming biased beliefs, the more biased these beliefs will be. How much the likely poor can expect to lose in consumption depends on the potential tax rates in different equilibria. Hence, the smaller is the difference in the potential policy outcomes, the more likely the POUM effect is. Specifically, if the voters do not think that their vote has an impact on the policy outcome, that is, if they do not act strategically, they always form the most optimistic beliefs possible and vote for low taxes. If the value of anticipation is low, individually and collectively rational choices diverge, and the poor voters are trapped in a bad equilibrium. By coordinating in voting for higher taxes, they could achieve higher welfare. In this case, the likely poor vote against their own self-interest.

The feasibility of the POUM effect also depends crucially on the specification of the cognitive technology, namely, on the naivete parameter χ. The less constraining the cognitive technology is, the more voters can bias their beliefs. Therefore, the POUM effect becomes more feasible as an explanation for the limited size of the government in democracies when we specify the cognitive technology with small values of χ. This can be clearly seen when comparing the results of Minozzi's (2013) POUM model with our results. In Minozzi's model voters are naive and can effectively choose their beliefs without the restrictions of prior beliefs or reality. When making the more conventional assumption about the voters forward-looking behavior and setting $\chi = 1$ corresponding to the standard Bayesian rationality in belief updating, the poor voters cannot bias their beliefs enough for the POUM effect to occur. This result, however, hinges on the simple specification with linear policy preferences and a policy choice between complete equalization and complete laissez-faire. By exogenously restricting the possible tax policies, it is shown that the POUM effect can be an important factor in voting behavior even if we endow the voters with a more realistic cognitive technology than in Minozzi (2013).

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Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1. Solve first the optimal recall rate given the constraint $\lambda < \frac{1}{2(1-q)}$. Note that here the constraint set is right-open. However the argument of the maximum is the lower and closed bound of the set and, hence, the maximum exists.

$$\begin{aligned}
\underline{\lambda} = & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \Big\{ \lambda \left[s[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] + \delta[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] \right] \\
& + (1-\lambda) \left[s[(1-\underline{\tau})[r(\lambda)y_H + (1-r(\lambda))y_L + \underline{\tau}\bar{y}] + \delta[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}] \right] \Big\} \\
= & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{ (1-\lambda) r(\lambda) \right\} \\
= & \underset{\lambda \in \left[0, \frac{1}{2(1-q)}\right)}{\arg\max} \left\{ \frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right\}
\end{aligned} \tag{23}$$

The derivative of the argument in (23) can be written as

$$\frac{d}{d\lambda} \left(\frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right) = -\frac{q^2}{[q + \chi(1-q)(1-\lambda)]^2} < 0 \tag{24}$$

and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\underline{\lambda} = 0$.

Solve the optimal recall rate given the constraint $\lambda \geq \frac{1}{2(1-q)}$.

$$\begin{aligned}
\overline{\lambda} = & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \Big\{ \lambda \left[s[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] + \delta[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] \right] \\
& + (1-\lambda) \left[s[(1-\overline{\tau})[r(\lambda)y_H + (1-r(\lambda))y_L + \underline{\tau}\bar{y}] + \delta[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] \right] \Big\} \\
= & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \left\{ (1-\lambda) r(\lambda) \right\} \\
= & \underset{\lambda \in \left[\frac{1}{2(1-q)}, 1\right]}{\arg\max} \left\{ \frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right\}
\end{aligned} \tag{25}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda} \left(\frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right) = -\frac{q^2}{[q + \chi(1-q)(1-\lambda)]^2} < 0 \tag{26}$$

and as before, is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\overline{\lambda} = \frac{1}{2(1-q)}$. Plugging in the optimal recall rates $\underline{\lambda}$ and $\overline{\lambda}$ into condition $U_{0,i}^{\underline{\lambda}} - U_{0,i}^{\overline{\lambda}} \geq 0$ and solving for s yields (13). □

Proof of Proposition 1. If $s \geq s^*$ the likely poor will choose the awareness rate $\lambda = 0$ and will not want to deviate by Lemma **??**. In this equilibrium, no one ever chooses

$\hat{\sigma}_i = y_L$, so the information set following this action is on off-equilibrium path and the beliefs in the information set following $\hat{\sigma}_i = y_L$ cannot be defined using Bayes rule or the variation of the Bayes rule or its variations. Define $p \equiv Pr[\sigma_i = y_H|\hat{\sigma}_i = y_L]$ and require $p \leq q$ to rule out the possibility of players strategically memorizing a low signal in order to end up with higher expectations. As the profitability of a deviation depends on whether the voters are able to increase their anticipatory utility by deviating, with these off-equilibrium path beliefs the likely rich have no incentive to deviate either. Given the strategies of the likely rich and the likely poor, the policy outcome as function of λ given in (8) implies $\tau^* = \underline{\tau}$.

If $s < s^*$, the likely poor choose the awareness rate $\lambda = \frac{1}{2(1-q)}$ and will not want to deviate by Lemma **??**. Given the strategies of the likely poor and the likely rich, the belief in the information set following $\hat{\sigma} = y_L$ is $Pr[\sigma = y_H|\hat{\sigma} = y_L] = 1$. Therefore, by deviating, a likely rich voter would end up believing to be likely poor and lose anticipatory utility. The likely rich have no incentive to deviate. The policy outcome as a function of λ given in (8) in this case implies $\tau^* = \overline{\tau}$. □

Proof of Proposition 2.

$$\frac{\partial s^*}{\partial \chi} = \frac{-\delta(\overline{\tau} - \underline{\tau})q\left[(1-\underline{\tau})\frac{\partial r(0)}{\partial \chi} - (1-\overline{\tau})(1-\overline{\lambda})\frac{\partial r(\overline{\lambda})}{\partial \chi}\right]}{[(1-\underline{\tau})r(0) - (1-\overline{\tau})(1-\overline{\lambda})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q]^2}, \tag{27}$$

where

$$\begin{aligned}&(1-\underline{\tau})\frac{\partial r(0)}{\partial \chi} - (1-\overline{\tau})(1-\overline{\lambda})\frac{\partial r(\overline{\lambda})}{\partial \chi}\\ &= (1-\overline{\tau})\frac{q(1-q)(1-\overline{\lambda})^2}{[q+\chi(1-q)(1-\overline{\lambda})]^2} - (1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} < 0\end{aligned} \tag{28}$$

since

$$\begin{aligned}&(1-\overline{\tau})\frac{q(1-q)(1-\overline{\lambda})^2}{[q+\chi(1-q)(1-\overline{\lambda})]^2} < (1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2}\\ &\iff (1-\overline{\tau})[q^2(1-\overline{\lambda})^2 + 2q\chi(1-q)(1-\overline{\lambda})^2 + \chi^2(1-q)^2(1-\overline{\lambda})^2]\\ &< (1-\underline{\tau})[q^2 + 2\chi q(1-q)(1-\overline{\lambda}) + \chi^2(1-q)^2(1-\overline{\lambda})^2]\end{aligned} \tag{29}$$

which holds since

$$q^2(1-\overline{\lambda})^2 + 2q\chi(1-q)(1-\overline{\lambda})^2 < q^2 + 2\chi q(1-q)(1-\overline{\lambda}) \tag{30}$$

and $1-\overline{\tau} < 1-\underline{\tau}$. Therefore $\frac{\partial s^*}{\partial \chi} > 0$.

□

Proof of Proposition 3.

$$\frac{\partial s^*}{\partial \overline{\tau}} = \frac{\delta q\big[(1-\underline{\tau})[r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda})]\big]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q]^2} \tag{31}$$

where

$$r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda}) = \frac{q^2}{[q+\chi(1-q)]2(1-q)[q+\chi\frac{1}{2}(1-2q)]} > 0. \tag{32}$$

Therefore $\frac{\partial s^*}{\partial \overline{\tau}} > 0$.

$$\frac{\partial s^*}{\partial \underline{\tau}} = -\frac{\delta q\big[(1-\overline{\tau})[r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda})]\big]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q]^2} \tag{33}$$

where

$$r(\underline{\lambda}) - (1-\overline{\lambda})r(\overline{\lambda}) = \frac{q^2}{[q+\chi(1-q)]2(1-q)[q+\chi\frac{1}{2}(1-2q)]} > 0. \tag{34}$$

Therefore $\frac{\partial s^*}{\partial \underline{\tau}} < 0$. □

Lemma 5 establishes a result that is useful in determining the sign of further partial derivatives of s^*.

Lemma 2. $(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) > 0$

Proof of Lemma 2.

$$\begin{aligned} &(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) \\ &= \frac{[2(1-q)(q+\chi\frac{1}{2}(1-2q))(1-\underline{\tau}) - (q+\chi(1-q))(1-2q)(1-\overline{\tau})]q}{(q+\chi(1-q))2(1-q)(q+\chi\frac{1}{2}(1-2q))}. \end{aligned} \tag{35}$$

Define

$$a \equiv 2(1-q)(q+\chi\frac{1}{2}(1-2q)), \tag{36}$$

$$b \equiv (q+\chi(1-q))(1-2q), \tag{37}$$

and write the numerator of (35) as

$$\begin{aligned} &[a(1-\underline{\tau}) - b(1-\overline{\tau})]q \\ \iff\ &[a-b-(a\underline{\tau} - b\overline{\tau})]q. \end{aligned} \tag{38}$$

The numerator of (35) is positive if

$$
\begin{aligned}
& a\underline{\tau} - b\overline{\tau} < a - b \\
\iff \; & a(1-\underline{\tau}) > b(1-\overline{\tau})
\end{aligned}
\tag{39}
$$

which holds since $a - b = q > 0$ and $\overline{\tau} > \underline{\tau}$ implies $1 - \underline{\tau} > 1 - \overline{\tau}$. The denominator of (35) is positive for all $q \in (0,1)$. Since both the denominator and the numerator of (35) are positive, the expression is positive and this establishes the result. □

Proof of Proposition 4. Write s^* as.

$$
s^* = \frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)}{\iota_{net}(\underline{\lambda}, \underline{\tau}) - \iota_{net}(\overline{\lambda}, \overline{\tau})} \tag{40}
$$

where

$$
\iota_{net}(\lambda, \tau) := \lambda[(1-\tau)y_L + \tau\bar{y}] + (1-\lambda)[(1-\tau)(r(\lambda)y_H + (1-r(\lambda))y_L) + \tau\bar{y}] \tag{41}
$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given λ and τ, and where $\underline{\lambda} = 0$ and $\overline{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to y_H holding the average income $\bar{y}$ constant.

$$
\frac{\partial s^*}{\partial y_H} = -\frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} < 0. \tag{42}
$$

By lemma 2, the derivative is negative for all parameter values, which implies that s^* decreases in y_H, when $\bar{y}$ is held constant.[17] Compute the partial derivative with respect to y_L holding the average income $\bar{y}$ constant.

$$
\frac{\partial s^*}{\partial y_L} = -\frac{\delta(\overline{\tau} - \underline{\tau})(y_H - \bar{y})[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau} - \underline{\tau})q]^2(\Delta y)^2} < 0 \tag{43}
$$

By lemma 2, the derivative is negative for all parameter values, which implies that s^* decreases in y_L, when $\bar{y}$ is held constant.[18] □

Proof of Proposition 5. Write s^* as.

$$
s^* = \frac{\delta(\overline{\tau} - \underline{\tau})(\bar{y} - y_L)}{\iota_{net}(\underline{\lambda}, \underline{\tau}) - \iota_{net}(\overline{\lambda}, \overline{\tau})} \tag{44}
$$

[17] By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^*}{\partial y_H} = -\frac{\bar{y} - y_L}{(y_H - \bar{y})^2}$, which is the result in Minozzi (2013).

[18] By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^*}{\partial y_L} = -\frac{1}{(y_H - \bar{y})}$, which is the result in Minozzi (2013).

where

$$\iota_{net}(\lambda,\tau) := \lambda[(1-\tau)y_L + \tau\bar{y}] + (1-\lambda)[(1-\tau)(r(\lambda)y_H + (1-r(\lambda))y_L) + \tau\bar{y}] \quad (45)$$

is the ex ante expectation of the expected net income of the likely poor in period 1 given λ and τ, and where $\underline{\lambda} = 0$ and $\overline{\lambda} = \frac{1}{2(1-q)}$. Compute the partial derivative with respect to $\bar{y}$ holding y_L and y_H constant.

$$\frac{\partial s^*}{\partial \bar{y}} = \frac{\delta(\overline{\tau}-\underline{\tau})[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda})(y_H - y_L)]}{[(1-\underline{\tau})r(0) - (1-\overline{\lambda})(1-\overline{\tau})r(\overline{\lambda}) - (\overline{\tau}-\underline{\tau})q]^2(\Delta y)^2} > 0 \quad (46)$$

By lemma 2, the derivative is positive for all parameter values, which implies that s^* increases in $\bar{y}$, when y_L and y_H are held constant.[19] □

Proof of Proposition 6. First, consider part (i). If $s^*(\chi) \geq s^{**}(\chi)$, then always when there is a low tax equilibrium, the likely rich are worse off in it. This condition is satisfied for $\chi = 1$ since $s^*(1) = s^{**}(1)$. This establishes part (i). Consider now part (ii). Show first that $s^*(\chi) < s^{**}(\chi)\ \forall \chi \in [0,1)$.

$$\begin{aligned}
& s^*(\chi) < s^{**}(\chi) \\
\iff & s^*(\chi) - s^{**}(\chi) < 0 \\
\iff & -(1-\underline{\tau})r(0) + \frac{1}{2}(1-\overline{\tau})r\left(\frac{1}{2(1-q)}\right) + q(\overline{\tau}-\underline{\tau}) < 0. \quad (47)
\end{aligned}$$

Now show that the left-hand side of (47) is increasing in χ. The derivative of the left-hand side of (47) with respect to χ is

$$\begin{aligned}
&(1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} - \frac{1}{4}(1-\overline{\tau})\frac{q(1-2q)}{[q+\frac{1}{2}\chi(1-2q)]^2} \\
>&(1-\underline{\tau})\frac{q(1-q)}{[q+\chi(1-q)]^2} - \frac{1}{4}(1-\underline{\tau})\frac{q(1-q)}{[q+\frac{1}{2}\chi(1-2q)]^2} \\
=&(1-\underline{\tau})q(1-q)\left[\frac{1}{[q+\chi(1-q)]^2} - \frac{1}{[2q+\chi(1-2q)]^2}\right], \quad (48)
\end{aligned}$$

where (48) is positive for all $\chi \in [0,1)$ since

$$\begin{aligned}
& [q+\chi(1-q)]^2 < [2q+\chi(1-2q)]^2 \\
\iff & [q+\chi(1-q)]^2 < [q+\chi(1-q)+(1-\chi)q]^2 \quad (49)
\end{aligned}$$

for all $\chi \in [0,1)$. Hence, the derivative of the left-hand side of (47) with respect to χ is positive so the left-hand side of (47) is increasing in χ. Since $s^*(1) = s^{**}(1)$, the

[19] By letting $\overline{\tau} = 1$, $\underline{\tau} = 0$, $\chi = 0$, and $\delta = 1$, we get $\frac{\partial s^*}{\partial \bar{y}} = \frac{y_H - y_L}{(y_H - \bar{y})^2}$, which is the result in Minozzi (2013).

left-hand side of (47) is zero when $\chi = 1$. Hence, since the left-hand side of (47) is increasing in χ, it has to be negative for $\chi \in [0, 1)$. This establishes that $s^*(\chi) < s^{**}(\chi)$ $\forall \chi \in [0, 1)$.

Now since $s^*(\chi) < s^{**}(\chi)$ $\forall \chi \in [0, 1)$, an existence of a low tax equilibrium does not necessarily mean that $s > s^{**}$ and the likely rich are worse off only if $s > s^{**}$. This establishes part (*ii*). □

Proof of Proposition 7. Denote the optimal choice of the likely poor by λ^*.

$$\begin{aligned}
\lambda^* &= \arg\max_{\lambda \in [0,1]} \Big\{ (1-\lambda)\left[s[(1-\tau)[r(\lambda)y_H + (1-r(\lambda))y_L] + \tau\bar{y}] + \delta[(1-\tau)y_L + \tau\bar{y}] \right] \\
&\quad + \lambda \left[(s+\delta)[(1-\tau)y_L + \tau\bar{y}] \right] \Big\} \\
&= \arg\max_{\lambda \in [0,1]} \big\{ (1-\lambda) r(\lambda) \big\} \\
&= \arg\max_{\lambda \in [0,1]} \left\{ \frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right\} \qquad (50)
\end{aligned}$$

The derivative of the argument can be written as

$$\frac{d}{d\lambda}\left(\frac{(1-\lambda)q}{q + \chi(1-q)(1-\lambda)} \right) = -\frac{q^2}{[q + \chi(1-q)(1-\lambda)]^2} < 0 \qquad (51)$$

and is always negative. The optimal recall rate is therefore given by the lower bound of the constraint, $\lambda^* = 0$. Since the maximum is unique, the choice $\lambda = \lambda^*$ stictly dominates all other choices of λ and, hence, the unique equilibrium is all the likely poor choosing λ^*.

□

Proof of Proposition 8. The utility of a representative likely poor voter is

$$U_0(0) = s[(1-\underline{\tau})[r(0)y_H + (1-r(0))y_L] + \underline{\tau}\bar{y}] + \delta[(1-\underline{\tau})y_L + \underline{\tau}\bar{y}], \qquad (52)$$

whereas if the likely poor coordinated on choosing $\lambda = \frac{1}{2(1-q)}$, a representative likely poor voter would enjoy utility

$$\begin{aligned}
U_0(\overline{\lambda}) &= \overline{\lambda}\big[s[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] + \delta[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] \big] \\
&\quad + (1-\overline{\lambda})\big[s[(1-\overline{\tau})[r(\overline{\lambda})y_H + (1-r(\overline{\lambda}))y_L] + \overline{\tau}\bar{y}] + \delta[(1-\overline{\tau})y_L + \overline{\tau}\bar{y}] \big], \qquad (53)
\end{aligned}$$

where $\overline{\lambda} = \frac{1}{2(1-q)}$. From Lemma 3, we know that if $s < s^*$, (53) is greater than (52). □